Chapter 4
Section 4.1

Activity
Suppose that a zombie outbreak has started. Each day a zombie can infect one more person with the zombie virus, turning them into a zombie as well. For the activity, also assume that no zombies get killed.

a.) On day 0 there is one zombie. On day 1 he infects another and there are now 2 zombies. How many zombies will there be on day 4?

$2^4 = 16$

b.) How many zombies will there be on day 7?

$2^7 = 128$

c.) Can you create a function that tells you how many zombies there are for each day?

On Day x there will be 2^x zombies.

d.) How will the above numbers be effected if a zombie can infect 2 people per day?

On Day x there will be 3^x zombies.

Further Questions: What happens if we start with more than one zombie? Is there a way to tell on what day a certain number of zombies has been reached?

Def: An Exponential Function with **base a** is a function of the form

$$a^x$$

where x is any real number and a is a real constant such that $a > 0$ and $a \neq 1$.

Q: Why can't $a = 1$?

Then we have constant function.

Q: Why can't $a < 0$?

Then a^x isn't defined everywhere.

Ex: Let $f(x) = 4^x$ and $h(x) = (\frac{1}{3})^x$. Evaluate $f(3/2)$ and $h(-2)$.

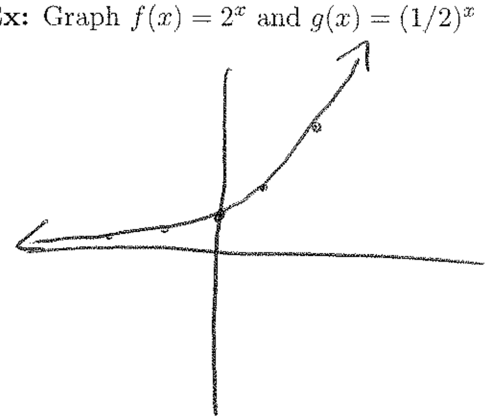
$f(3/2) = 4^{3/2} = 2^3 = 8$ $\qquad$ $h(-2) = (\frac{1}{3})^{-2} = 3^2 = 9$

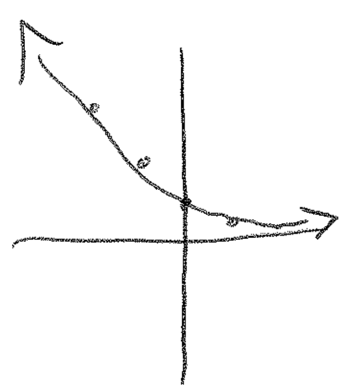
Domain of Exponential Functions
The domain of $f(x) = a^x$ for $a > 0$ and $a \neq 1$ is the set of all real numbers, $\mathbb{R}$

Q: How do we get $2^{\sqrt{3}}$?

Approximate with rational exponents.

Graphing Exponential Functions
Ex: Graph $f(x) = 2^x$ and $g(x) = (1/2)^x$





Properties of Exponential Functions
The exponential function $f(x) = a^x$ has the following properties:

i) The function f is increasing for $a > 1$ and decreasing for $0 < a < 1$.

ii) The y-intercept of the graph of f is $(0, 1)$.

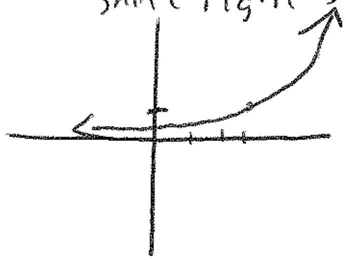
iii) The graph has the x-axis as a horizontal asymptote.

iv) The domain of f is $(-\infty, \infty)$ and the range of f is $(0, \infty)$

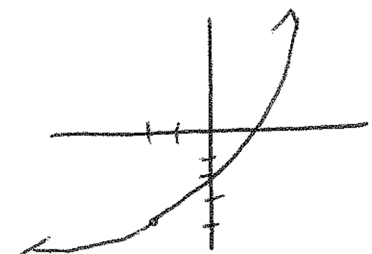
v) The function f is one-to-one.

Exponential Transformations
If $f(x) = a^x$ is an exponential function, then the function $g(x) = b \cdot a^{x-h} + k$ is a transformation of the graph of f.
Ex: Graph $y = 2^{x-3}$ and $f(x) = -4 + 3^{x+2}$.

Shift right 3



Down 4, left 2



Exponential Equations
Because exponential functions are one-to-one we have that: For $a > 0$ and $a \neq 1$

$$\text{if } a^{x_1} = a^{x_2}, \text{ then } x_1 = x_2$$

Ex: Solve $4^x = \frac{1}{4}$ and $(\frac{1}{10})^x = 100$

$x = -1$ $\quad$ $x = -2$

Compound Interest
Similar to the zombie problem, if money is left in a bank, your interest will earn interest and so on so that the longer the money stays the more you earn.
Formula: If a principal P is invested for t years at an annual rate r compounded n times per year, then the amount A, or ending balance, is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Ex: Find how much money you have in the bank after 3 years if you started with 20,000 with an interest rate of 6% compounded daily.

$$A = 20{,}000\left(1 + \frac{.06}{365}\right)^{3 \cdot 365}$$

Q: What would happen if you compound continuously?
Def: $e = \lim_{n\to\infty}(1 + \frac{1}{n})^n \approx 2.718281828459$
Practice: 22, 28, 32, 36, 42, 48, 52, 64, 73

$$A = 20{,}000\, e^{(.06)3}$$